We have a continuous family of curves between the curve for two tanks and that for one tank. The curves for low R approach zero more quickly, as predicted. The curves dip downward from zero slope sooner as R increases, and the inflection points move up the curve.

APPLICATION OF THE MODEL

The model is most useful for cases where the vessel being studied would correspond to a series of tanks with a low value of n but where data lie between the curves for two discrete values of n. If the corresponding number of tanks in the series is high (let us say, about ten), it would be sufficiently accurate to take the next higher or lower n. If the value falls between one and two, two and three, three and four, and so forth, one would choose the higher value of n and then select a value of R to fit the experimental data.

SUMMARY

A two parameter model for a nonideal flow reactor was presented. The equations for the residence time distribution curves were derived. The advantages of this model were described.

NOTATION

C = concentration
F = volumetric flow rate
R = recycle ratio
t = time
T = reduced time, t/θ
V = volume of one tank in the series
f = transient response function
i = any given tank in series
n = number of tanks in series
$u(t)$ = unit step function
θ_s = normal holding time of the system (V/F)

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Magnetohydrodynamics of Liquid Films Flowing along a Vertical Plane Surface

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The problem of why when a vessel of liquid has been emptied and set aside, a thin film of liquid clings to the inside and gradually drains down to the bottom under the action of gravity, was posed and solved by Jeffreys (*1*). The problem, which has wide practical applications, has been reexamined very recently by Gutfinger and Tallmadge (*2*), who included the unsteady term, and by Hassan and Franzini (*3*), who included the inertia terms without the unsteady term in the equation of motion. In this problem of drainage we want to know the flow rate and the thickness of the liquid film draining down a surface. These variables depend upon the shape of the surface; the surface tension; and the gravitational, inertial, viscous, and other external forces if any. A study of the effect of an external force like $\vec{J} \times \vec{B}$ arising due to the interaction of electromagnetic forces and hydrodynamic forces on the viscous lifting of a conducting fluid film has been made by Bradshaw et al. (*4*).

We will consider here the effect of the hydromagnetic force $\vec{J} \times \vec{B}$ on the thickness of the draining liquid film of conductivity σ along a vertical plane surface when the unsteady term is also taken account of in the equation of motion. The inclusion of the unsteady term can be justified because steady conditions are impossible to obtain when film thickness changes continuously with time. On the assumption that the force $\vec{J} \times \vec{B}$ simplifies to $-\sigma B_o^2 u$, it is seen that the equations of the problem are identical to those for one-dimensional unsteady heat conduction with heat generation linear with temperature, as pointed out by the reviewer.

With the usual assumption the equation of fluid flow past a vertical plane surface in the presence of a magnetic field B_o imposed in the y direction is

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g - \frac{\sigma B_o^2 u}{\rho} \qquad (1)$$

and the equation of continuity is

$$-\frac{\partial h}{\partial t} = \frac{\partial}{\partial x} \int_o^h u \, dy \qquad (2)$$

where $\nu = \mu/\rho$ is the kinematic viscosity and $h(x,t)$ is the film thickness.

The boundary and initial conditions are

$$\left.\begin{array}{ll} u(y,o) = 0, & u(o,t) = 0, \\ \dfrac{\partial u}{\partial y} = 0 & \text{at } y = h \end{array}\right\} \qquad (3)$$

and

$$h(o,t) = 0 \qquad (4)$$

The solution of Equation (1) with conditions (3) has been given by Carslaw and Jaeger (5) and is

$$u(y,t) = \frac{g}{\nu M^2}\left[I - \frac{\cosh M(y-h)}{\cosh Mh} \right] - \frac{16\, gh^2}{\pi\nu}\sum_{n=1}^{\infty} \frac{e^{-\left(M^2 + \frac{\lambda_n^2\pi^2}{4h^2}\right)\nu t}}{\lambda_n(\lambda_n^2\pi^2 + 4\, M^2h^2)} \times \sin\frac{\lambda_n \pi y}{2h} \quad (5)$$

where $M^2 = \sigma B_o^2/(\rho\nu)$, $\lambda_n = 2n - 1$.

Integrating (5) over the film thickness gives the flow rate

$$q = \int_0^h u\, dy = \frac{g}{\nu M^2}\left(h - \frac{\tanh Mh}{M} \right) - \frac{32\, g\, h^3}{\pi^2\nu}\sum_{n=1}^{\infty} \frac{e^{-\left(M^2 + \frac{\lambda_n^2\pi^2}{4h^2}\right)\nu t}}{\lambda_n^2\,(\lambda_n^2\pi^2 + 4\, M^2h^2)} \quad (6)$$

Combining (6) with (2) and (4) we get

$$x = \frac{gt \tanh^2 Mh}{\nu M^2} + \frac{64\, g\, h^2}{\pi^2\nu^2}\sum_{n=1}^{\infty} \frac{e^{-\left(M^2 + \frac{\lambda_n^2\pi^2}{4h^2}\right)\nu t}}{\lambda_n^2(\lambda_n^2\pi^2 + 4\, M^2h^2)^3} [10\, \lambda_n^2\pi^2h^2 + \lambda_n^4\pi^4\nu t + 4h^2M^2(2h^2 + \lambda_n^2\pi^2\nu t)] \quad (7)$$

The limiting case $M \rightarrow 0$ has been found to agree with the results given by Gutfinger and Tallmadge (2).

Further, for large values of time, the flow rate and the film thickness become

$$q = \frac{g}{\nu M^2}\left(h - \frac{\tanh Mh}{M} \right) \quad (8)$$

and

$$x = \frac{g\, t}{\nu}\left(\frac{\tanh Mh}{M} \right)^2 \quad (9)$$

Identical results are obtained for large values of M.

Since from (9), $dh/dM = (\sinh 2Mh - 2Mh)/2M^2$ and is positive for all M and h, the film thickness h increases with the increase in M for any given x and t. Therefore, it may be concluded that the fall of the film can be delayed by using hydromagnetic effects.

NOTATION

B_o = applied magnetic field in the y direction
g = gravitational acceleration
h = thickness of the liquid film
M = $B_o(\sigma/(\rho\nu))^{1/2}$
q = flow rate per unit width
t = time
u = velocity in the x direction
x = vertical coordinate in the downward direction
y = coordinate measured along the perpendicular to the plane of the wall

Greek Letters

μ = dynamic viscosity
ρ = density
ν = kinematic viscosity
σ = electrical conductivity

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A Heat Transfer Analogy for Diffusion and First-Order Chemical Reaction in a Catalyst Pore

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The expressions for rate of reaction in a catalyst pore can be expressed neither in terms of simple additive resistances nor by means of a simple kinetic rate law proportional to catalyst surface area. However, a rather thorough-going analogy exists between a first-order reaction in a catalyst pore and heat transport to a surrounding fluid from a longitudinal fin on a heated tube. This analogy extends not only to catalytic and heat transfer efficiency, but also to the nonlinear dependence of rate on transfer area and to the correspondence that exists between catalyst poisoning and fouling of a heat transfer surface.

For an idealized cylindrical catalyst pore the usual boundary conditions rest upon taking the fluid composition at the pore mouth equal to that of the bulk fluid phase, C_1 and assuming zero concentration gradient at the bottom of the pore. The steady state concentration distribution for this situation is

$$\frac{C(x)}{C_1} = \frac{\cosh [\lambda_c(L - x)]}{\cosh (\lambda_c L)} \quad (1)$$